

# An Improved Sobel Edge Detection

Wenshuo Gao

Digital Media Department, Communication University of  
China  
ICICT 2010  
Beijing, China  
gws8620@sina.com

Xiaoguang Zhang

Digital Media Department, Communication University of  
China  
ICICT 2010  
Beijing, China  
zxgprc1988@yahoo.cn

Lei Yang

Digital Media Department, Communication University of  
China  
ICICT 2010  
Beijing, China  
young-lad@263.net

Huizhong Liu

Digital Media Department, Communication University of  
China  
ICICT 2010  
Beijing, China  
liujie1621@163.com

**Abstract**—This paper proposes a method which combines Sobel edge detection operator and soft-threshold wavelet de-noising to do edge detection on images which include White Gaussian noises. In recent years, a lot of edge detection methods are proposed. The commonly used methods which combine mean de-noising and Sobel operator or median filtering and Sobel operator can not remove salt and pepper noise very well. In this paper, we firstly use soft-threshold wavelet to remove noise, then use Sobel edge detection operator to do edge detection on the image. This method is mainly used on the images which includes White Gaussian noises. Through the pictures obtained by the experiment, we can see very clearly that, compared to the traditional edge detection methods, the method proposed in this paper has a more obvious effect on edge detection.

**Key words:** *soft-threshold wavelet de-noising; edge detection operator; White Gaussian noises; edge detection*

## I. INTRODUCTION

Using computers to do image processing has two objectives: First, create more suitable images for people to observe and identify. Second, we wish that computers can automatically recognize and understand images. The edge of an image is the most basic features of the image. It contains a wealth of internal information of the image. Therefore, edge detection is one of the key research works in image processing.

The current image edge detection methods are mainly differential operator technique and high-pass filtration. Among these methods, the most primitive of the differential and gradient edge detection methods are complex and the effects are not satisfactory. The widely used operators such as Sobel, Prewitt, Roberts and Laplacian are sensitive to noises and their anti-noise performances are poor. The Log and Canny edge detection operators which have been proposed use Gaussian function to smooth or do convolution

to the original image, but the computations are very large. This paper mainly used the Sobel operator and soft-threshold wavelet de-noising method to do edge detection processing on the images which have been disturbed by white Gaussian noises. It has been proved that the effect by using this method to do edge detection is very good and its anti-noise performance is very strong too. accuracy.

## II. THE PRINCIPLE OF EDGE DETECTION

In digital image, the so-called edge is a collection of the pixels whose gray value has a step or roof change, and it also refers to the part where the brightness of the image local area changes significantly. The gray profile in this region can generally be seen as a step. That is, in a small buffer area, a gray value rapidly changes to another whose gray value is largely different with it. Edge widely exists between objects and backgrounds, objects and objects, primitives and primitives. The edge of an object is reflected in the discontinuity of the gray. Therefore, the general method of edge detection is to study the changes of a single image pixel in a gray area, use the variation of the edge neighboring first-order or second-order to detect the edge. This method is used to refer as local operator edge detection method. Edge detection is mainly the measurement, detection and location of the changes in image gray. Image edge is the most basic features of the image. When we observe the objects, the clearest part we see firstly is edge and line. According to the composition of the edge and line, we can know the object structure. Therefore, edge extraction is an important technique in graphics processing and feature extraction.

The basic idea of edge detection is as follows: First, use edge enhancement operator to highlight the local edge of the image. Then, define the pixel "edge strength" and set the threshold to extract the edge point set. However, because of the noise and the blurring image, the edge detected may not be continuous. So, edge detection includes two contents.

First is using edge operator to extract the edge point set. Second is removing some of the edge points from the edge point set, filling it with some another and linking the obtained edge point set into lines.

### III. AN EDGE DETECTION MODEL BASED ON SOBEL OPERATOR

Compared to other edge operator, Sobel has two main advantages: ①Since the introduction of the average factor, it has some smoothing effect to the random noise of the image. ②Because it is the differential of two rows or two columns, so the elements of the edge on both sides has been enhanced, so that the edge seems thick and bright.

In the airspace, edge detection is usually carried out by using the local operator. What we usually use are orthogonal gradient operator, directional differential operator and some other operators relevant to second-order differential operator. Sobel operator is a kind of orthogonal gradient operator. Gradient corresponds to first derivative, and gradient operator is a derivative operator. For a continuous function  $f(x, y)$ , in the position  $(x, y)$ , its gradient can be expressed as a vector (the two components are two first derivatives which are along the X and Y direction respectively):

$$\nabla f(x, y) = [G_x \quad G_y]^T = \left[ \frac{\partial f}{\partial x} \quad \frac{\partial f}{\partial y} \right] \quad (1)$$

The magnitude and direction angle of the vector are:

$$mag(\nabla f) = |\nabla f| = \left[ G_x^2 + G_y^2 \right]^{1/2} \quad (2)$$

$$\phi(x, y) = \arctan \left( \frac{G_x}{G_y} \right) \quad (3)$$

The partial derivatives of the formulas above need to be calculated for each pixel location. In practice, we often use small area template convolution to do approximation.  $G_x$  and  $G_y$  need a template each, so there must be two templates combined into a gradient operator. The two  $3 \times 3$  templates used by Sobel are showed as (a) and (b). Every point in the image should use these two kernels to do convolution. One of the two kernels has a maximum response to the vertical edge and the other has a maximum response to the level edge. The maximum value of the two convolutions is used as the output bit of the point, and the result is an image of edge amplitude.

-1	-2	-1
0	0	0
1	2	1

(a) Convolution template S1

-1	0	-1
-2	0	2
-1	0	1

(b) Convolution template S2

Figure 1. Sobel edge operator

Their convolution is as follows:

$$g_1(x, y) = \sum_{k=-1}^1 \sum_{l=-1}^1 S_1(k, l) f(x+k, y+l) \quad (4)$$

$$g_2(x, y) = \sum_{k=-1}^1 \sum_{l=-1}^1 S_2(k, l) f(x+k, y+l) \quad (5)$$

$$g(x, y) = g_1^2(x, y) + g_2^2(x, y) \quad (6)$$

If  $g_1(x, y) > g_2(x, y)$ , it means that there is an edge with a vertical direction passing through the point  $(x, y)$ . otherwise, an edge with a level direction will pass through the point. If the pixel value of the point  $(x, y)$  is  $f(x, y)$ , and this point is judged as an edge point if  $f(x, y)$  Satisfy one of the following two conditions.

$$1) (1) \quad g(x, y) > 4 \times \frac{\sum_{i=1}^{row} \sum_{j=1}^{list} g^2(i, j)}{row \times list}$$

$$(2) \quad g_1(x, y) > g_2(x, y)$$

$$(3) \quad g(x, y-1) \leq g(x, y)$$

$$(4) \quad g(x, y) \geq g(x, y+1)$$

$$2) (1) \quad g(x, y) > 4 \times \frac{\sum_{i=1}^{row} \sum_{j=1}^{list} g^2(i, j)}{row \times list}$$

$$(2) \quad g_1(x, y) > g_2(x, y)$$

$$(3) \quad g(x-1, y) \leq g(x, y)$$

$$(4) \quad g(x, y) \leq g(x+1, y)$$

In the formulas above, row and list refer to the number of rows and columns of the image respectively.

### IV. THE PRINCIPLE OF WAVELET THRESHOLD DE-NOISING

Those traditional methods which use the base function with infinite width (for example, Fourier transform uses sinusoidal curvelet as its orthogonal basis function) exist many flaws. In order to overcome these shortcomings, mathematicians and engineers have developed kinds of methods which use the basis functions with finite width to do transformation. These basis functions change not only in frequency but also in the position. Their width is limited and they are known as the wavelet. The transformations Based on them are called wavelet transform.

The reason why wavelet de-noising method is successful is that wavelet transform has the following important features:

- (1) The nature of low-entropy: The sparse distribution of wavelet coefficients makes the entropy of the transformed signal reducing.
- (2) The nature of multi-resolution: By adopting the approach of multi-resolution, we can describe the non-stationary characteristics of the signal (such as

edge, peak, breakpoint and so on) very well in order to extract and protect the feature.

- (3) The nature of de-correlation: The wavelet transform can be de-related to the signal and the noise tends to be whitening after transformation, so it is more conducive to de-noising in the wavelet domain than in the time domain.
- (4) The diversity nature of wavelet selection: The wavelet transform can select transform radix flexibly. So we can choose different wavelet functions for different applications in order to get the best treatment effect.

In recent years, with the application and development of wavelet, it has been well used in signal processing, image processing, quantum theory, and other technology fields. Because the wavelet transform can select transform radix flexibly, it can select multi-band wavelet, wavelet packet, translation invariant wavelet and so on according to the signal characteristics and the requirements of signal de-noising. On different occasions, we can choose different mother wavelets.

Image de-noising processing is actually an image filter processing. The principle of Wavelet transform for image filtering is: Suppose  $\{V_j\}_{j \in z}$  is a multi-resolution analysis of  $L^2(R)$ ,  $W_j$  is the orthogonal complement space of  $V_j$  in  $V_{j+1}$ . Make  $P_j$  and  $Q_j$  be the orthogonal projection of  $L^2(R)$  to  $V_j$  and  $W_j$  respectively. Suppose there is a signal  $f(t) \in L^2(R)$ , the signal with noise measured by instruments is  $P_j f \in V_j$ , thus:

$$P_j f = P_j - n f + \sum_{j=1}^{j-1} Q_j f \quad (7)$$

The process of signal decomposition with wavelet transform is the process that separates the various frequency components into different frequency bands from high to low gradually.

Because of the linear nature of the wavelet transform, after do discrete wavelet transform to the observed signal, the wavelet coefficient obtained is still consisted of two parts: One part is the wavelet coefficient corresponding to the signal; the other is corresponding to the noise. To the wavelet coefficient of noisy signals, if it is greater than the specified threshold value, it is considered to have the signal component, this is the result caused by both the signal and the noise and it will be retained. If it is less than the threshold value, it is considered not to have the signal component, it is just the result of the noise and filter the coefficients like this can get the effect of noise reduction. This is the principle of wavelet threshold de-noising. Wavelet threshold filtering method can be carried out according to the following basic steps: (1) Do wavelet transform to the noisy image; (2) determine the filtering threshold value of each wavelet; (3) the selection of threshold function; (4) wavelet inverse

transform; (5) observe the de-noised image, if the result is good then end, or else return to Step 1 or Step 2.

Wavelet threshold de-noising algorithm includes hard-threshold algorithm and soft-threshold algorithm, and the expression of hard-threshold algorithm is:

$$\hat{W}_{j,k} = \begin{cases} W_{j,k}, & |W_{j,k}| \geq \lambda \\ 0, & |W_{j,k}| < \lambda \end{cases} \quad (8)$$

$$\hat{W}_{j,k} = \begin{cases} \text{sign}(W_{j,k}) \cdot (|W_{j,k}| - \lambda), & |W_{j,k}| \geq \lambda \\ 0, & |W_{j,k}| < \lambda \end{cases} \quad (9)$$

This paper mainly uses Soft-threshold method to do filtering to the images which contain Gaussian white noises.

## V. THE IMPROVED ALGORITHM

The advantage of Sobel edge operand is its smoothing effect to the random noises in the image. And because it is the differential separated by two rows or two columns, so the edge elements on both sides have been enhanced and make the edge seems thick and bright. Sobel operator is a gradient operator. The first derivative of a digital image is based on a variety of two-dimensional gradient approximation, and generates a peak on the first derivative of the image, or generates a zero-crossing point on the second derivative. Calculate the magnitude and the argument value of the image horizontal and vertical first-order or second-order gradients, at last calculate modulus maxima along the angular direction and obtain the edge of the image. But when the image has lots of white Gaussian noises, it is very difficult to get the peak value of the first derivative, the reason is because that the noise points and the useful signals mix up. Therefore this paper combines Sobel operator and soft-threshold wavelet de-noising. The core idea of the algorithm is:

- (1) Do wavelet decomposition to the image matrix and get the wavelet coefficients with noises.
- (2) Process the wavelet coefficients HL, LH and HH obtained by the decomposition, and keep the low-frequency coefficients unchanging.
- (3) Select an appropriate threshold to remove Gaussian white noise signals.
- (4) Do inverse wavelet transformation to the image matrix and get the image matrix after de-noising.
- (5) Custom template edge coefficient according to the Sobel operator template showed in Figure 1.
- (6) After given Sobel edge detection operator template, convolute on every pixel of the image using this template, get the gradient of this point, and the gradient amplitude is the output of this point. At last we get the edge detection image.

## VI. EXPERIMENTAL RESULTS AND ANALYSIS

This paper will use a Lena image with Gaussian white noise as the original image. First, use the traditional edge detection operators (include Sobel operator, Prewitt operator, Laplacian operator and Canny operator) to do edge detection

to the noisy image. Then, use the commonly used methods which combine mean de-noising and Sobel operator or median filtering and Sobel operator. However, these methods can not remove salt and pepper noise very well. At last, the paper proposes the Sobel edge detection operator based on soft-threshold wavelet de-noising. When doing edge detection to the image with Gaussian white noise signals, it can remove the noise effectively and can detect the edge detail very well.

Use the Lena image with Gaussian white noise as the original image. First, use the traditional edge detection operators to do edge detection, and the results are showed in Figure 2,3,4,5,6:



Figure 2. the original Lena image



Figure 3. Sobel edge detection operator



Figure 4. Prewitt edge detection operator

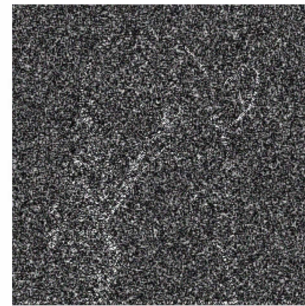


Figure 5. Laplacian edge detection

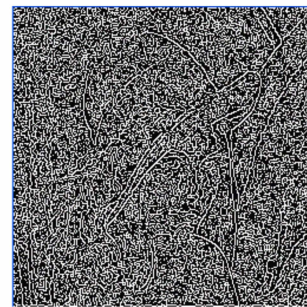


Figure 6. Canny edge detection operator

From the figures above we can see that after adding Gaussian white noises to the image, using the traditional operators to do edge detection to the images will detect all the noise points and will also blur the edge details of the images. Even the detection result of the classical Canny operator is unsatisfactory either. This is because that the traditional edge detection operators mostly use the differences of the neighborhood gray values. The first-derivative's extreme range of the adjacent pixels' edge will change obviously, so they can detect image edge. However, when the image adulterates lots of noise signals, there are gray value differences between white noises and image signals, and they can be detected easily. This leads to the poor detection effect of the classical operators.

In order to overcome this defect, the paper combines some commonly used de-noising methods and these classical operators, such as median filter and mean filter de-noising, see in Figure 7 and 8:

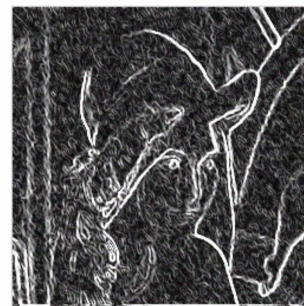


Figure 7. median filter and Sobel operator



Figure 8. mean filter and Sobel operator

Figure 7 shows the method which combines the median filter and Sobel operator. Median filter can remove spike and high-frequency noise signals, it can also suppress step and slope mutation, at the same time it doesn't cause mutations in phase. Therefore, it plays a certain role in filtering white noise signals. We can see that compared the result with that obtained by other classical operators, the noise signals of the image are removed cleaner. However, it is just due to setting high-frequency coefficients to 0 leads to the reduction and removing of peak hopping when doing first-order derivative to the image edge. And this makes the images obtained by edge detection very blurring and it will also neglect some edge details.

Figure 8 shows the method which combines mean filter and Sobel operator. This method does weighted average to the signal in the process of de-noising. It improves the SNR effectively, makes the useful signal more obvious and plays a certain role in the smoothing filter. But this method is not filtering the noise signals truly. It is just reducing the noise signals. Therefore, the result detected is still unsatisfactory.

The paper proposes an edge detection method which combines soft-threshold wavelet de-noising and Sobel operator. See in Figure 9:



Figure 9. soft-threshold wavelet de-noising and Sobel operator

Figure 9 shows the method which combines wavelet de-noising and Sobel operator. The advantage of this method is that it can maximumly filter the Gaussian white noises focused in the high-frequency parts of the signal. From the Figure 9 we can see that this method can remove noise signals very effectively and it can also detect the edge of the image very well. Compared the result with those obtained by the classical operators, the image noise signals are filtered

cleaner. Compared with Figure 8 and 9, wavelet de-noising has a more obvious de-noising nature, this makes the effect of edge detection using Sobel operator more outstanding. This algorithm is proposed for the edge detection of the images with Gaussian white noises.

#### ACKNOWLEDGMENT

This work is supported by the "211" Key Subject Construction Project of Communication University of China. (21103050102)

#### REFERENCES

- [1] Sobototkka, K. Pitas. I. Looking for Faces and Facial Feature in Color Images, Pattern Recognition and Image Analysis, 7(1): 1997
- [2] Li, H. Forchheimer. R. Location of face using color cues. Picture Coding Symp., Lausanne, Switzerland, Mar, 1993,2-4
- [3] Kenneth R.Castleman. Digital Image Processing. Beijing : Tsinghua university press, 1998
- [4] Ostu N.A. Threshold Selection Method from Gray - Level Histograms[J ].IEEE Trans on Systems. Man and Cybernetics, SMC - 9,1979,9(1):62-66
- [5] Chai, D.Ngan. K.N. Face Segmentation Using Skin-Color Map In Videophone Application, IEEE Trans. Circuits and System for Video Technology, 9(4): June 1999
- [6] William Ng, Jack Li, Simon Godsill. A review of recent results in multiple target tracking[C ] // Proc. Of the IEEE International Conference on Robotics and Automation Orlando, Florida, 2006: 3807-3812
- [7] Sanjeev Arulampalam, Simon Maskell, Neil Gordon. A tutorial on particle filters for online non-linear/non-gaussian Bayesian tracking[J ]. IEEE Trans. On Signal Processing, 2002, 50(2): 174-188
- [8] Azzalini A, Farge M, Schneider K. Nonlinear wavelet threshold : A recursive method to determine the optimal de-noising threshold[J ]. Appl. Comput. Harmon. Anal , 2005,18: 177-185
- [9] Donoho D L. De-noising by soft-threshold [J ] . IEEE Trans .on Information Theory, 1995,41: 613-627
- [10] Chang S G, Bin Yu, Vetlerti M. Adaptive wavelet threshold for image de-noising and compression [J ]. IEEE Trans Image Processing, 2000, 9(9):1532-1546